Reepham Primary School Calculation Policy

December 2020

Introduction

This calculation policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). It provides guidance on appropriate calculation methods and progression throughout the school.

Mental calculation

1. Of greatest importance in the real world is the ability to use a range of mental methods for calculation and estimation. These may be supported by jottings.
2. The prerequisites of mental calculation are an understanding of place value and knowledge of key strategies such as partitioning, building number facts, finding complements, rounding and adjusting, doubling and halving etc.
3. We do not rely on memorisation alone, but aim for fluency with number bonds and multiplication facts alongside systematic refinement of methods of operating on numbers.
4. Our aim is that pupils acquire a repertoire of mental methods and are able to make confident decisions about which strategy they choose to use for particular numbers.
5. It is important that when faced with a calculation that pupils ask themselves if they can solve it mentally or if a written formal method is required.

Written calculation

1. Due to the availability of calculators, written methods are no longer used as a regular part of technical jobs or for most day-to-day tasks. As a result, practice of written methods is reduced and connection to mental methods has become more important.
2. A 'formal' method applies the same approach for any numbers and is recorded using a vertical format. Pupils need such a method because it gives them a 'fail safe' strategy. It is more important for this method to be accurate and fluent than it is for the recording to be compact.
3. It aids consistency if we agree a formal method for each operation, and ensure that this method develops systematically from our progression in mental calculation. At a later stage, exploring different written methods can enrich pupils' experience.
4. Our aim is that pupils acquire a formal method for each of the four operations and are able to apply these when appropriate, and always including a mental estimate and a checking strategy.

Web links suggested resources

- NCETM website for examples and activities.
- Nrich website
- Testbase
- https://uk.ixl.com/ - for questions on any subject:
- http://www.sheffieldmaths.co.uk/
- www.stem.org.uk
- Progression in Models and Images booklets in Public – Staff Files – Curriculum Subjects – Maths - Maths Resources
- Topmarks for the use of resources and ITP's.
- Woodlands junior homework
- BBC Bitesize
- http://www.multiplication.com/
- My Maths
- Sumdog
- TES website
- http://www.amathsdictionaryforkids.com/
- http://www.teachingideas.co.uk/subjects/maths
- http://www.maths-games.org/fraction-games.html

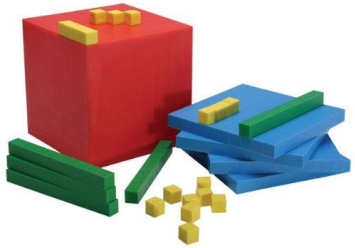
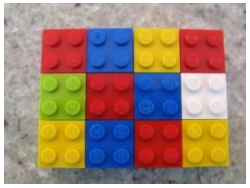
Representations

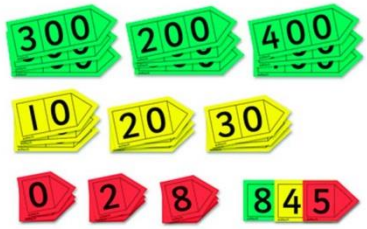
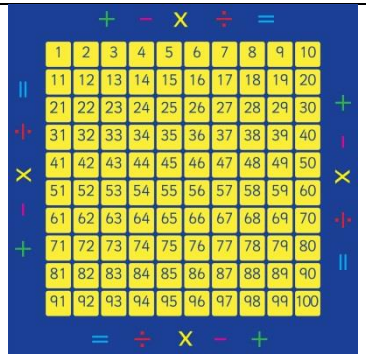
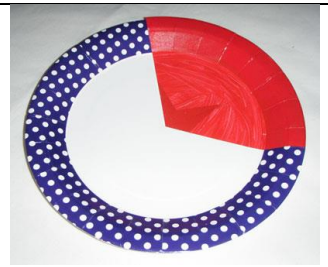
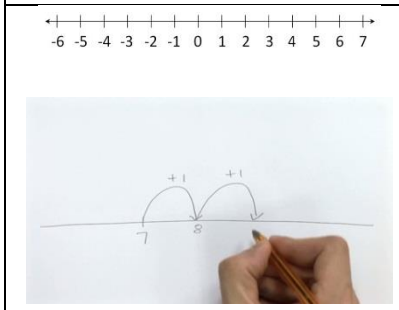
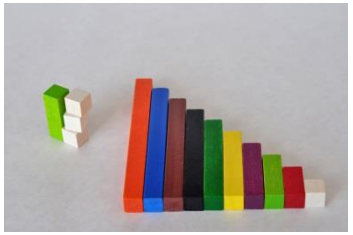
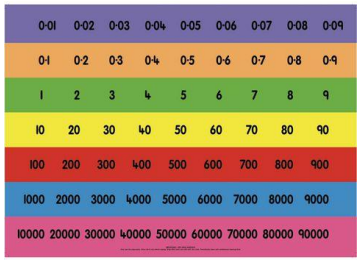
Representations are vitally important throughout a child's maths education. Representations provide a 'hook' for children to 'hang' mathematical concepts, and allow children to manipulate and later visualise the structure of mathematics.

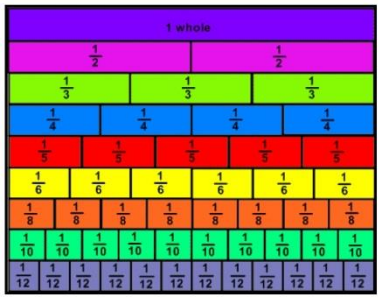
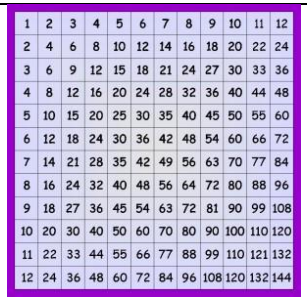
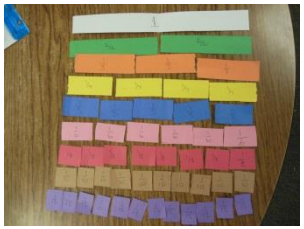
Representations are therefore a significant aid in developing conceptual understanding.

Different concepts can be represented using the same resource/representation depending on the child's age and stage of mathematical development.

Here are some of the key representations that will be in use throughout a child's maths education in our school.

 Counters	 Dienes	 Multilink	 Arrays	 Bead String

 Numicon	 Arrow cards	 Hundred Square	 Fraction/Decimal /Percentage cubes	 Fraction Plate
 Number lines	 Counting Stick	 Coins	 Cuisenaire rods	 Place value chart

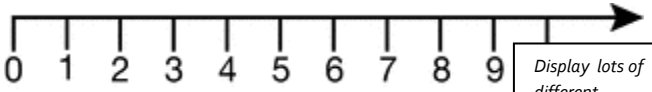
				
Fraction wall	Multiplication Square	MMS cups	Fraction Kit	

Place value and counting

Stage 1

Place value and ordering

Read and write numbers 0 to 10 then 0 to 20. Match numbers to corresponding number of objects. Order and visualise them on a number line (e.g. hanging digit cards) and use related language, more than, less than (fewer), etc.



Display lots of different representations

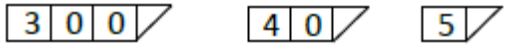
Build number language in a structured way to 999, e
place value (pv) chart:

1 2 3 4 5 6 7 8 9
10 20 30 40 50 60 70 80 90
100 200 300 400 500 600 700 800 900

Work first on numbers with regular language:

- hundreds, then combine H + U, e.g. '600 and 7'
- 40, 60, 70, 80, 90, then combine into H+T+U, e.g. 'five hundred and seventy two'
- 20, 30, 50, then 10 to 19, then all combinations

Develop writing and partitioning numbers to 999, e.g. use arrow cards, 345 = 300 + 40 + 5:



Relate the value of each digit to the pv chart, e.g. '4' is forty or 4 tens.

Counting and properties of numbers

Count forward and back to and from 100, starting at any number.
Extend to counting in tens (e.g. on 100 square) and hundreds (e.g. on counting stick).
Starting from zero, count in steps of 2 and 5.
Work with small collections of objects in regular and irregular arrangements. Recognise quantities up to at least 5 and count systematically for larger quantities.
Begin to write and spell numbers in words as well as figures.

Stage 2

Place value and ordering

Read, write and order numbers to 1000 and beyond, and partition into hundreds, tens and ones (e. g. use place value chart, arrow cards and denes).
Use funny voice when speaking in Maths – thousands = deep, slow voice hundreds = high, quick voice.

Compare any two 3-digit numbers, say which is more or less (using the signs >, <), and give a number between them.

Visualise numbers, e.g. their position on the number line or a 100 square, and read simple scales.

Identify the multiple of 10 or 100 that is nearest to any number.

Counting and properties of numbers

Secure counting, forwards and backwards and develop the language of ordinal numbers.
Count on and back from any number in steps of different size, noting patterns in the sequences:

- steps 2, 5, 10 and 100
- steps of ½ and ¼
- work particularly on sequences of multiples
- for steps of 2, use the language of odd and even numbers

Estimate and then count a set of objects reliably in ones and twos.
Write and spell two digit numbers in words accurately.

Stage 3

Place value, ordering and rounding

By extending the place value chart, develop the language of numbers to 999 999:

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000
100 000	200 000	300 000	400 000	500 000	600 000	700 000	800 000	900 000

Work first on the language of the new rows:

- read across rows, establishing the word 'thousand', e.g. 5 000, 70 000 and 800 000
- combine numbers from the new rows e.g. 'six hundred and fifty seven thousand'
- combine numbers from each row, e.g. 'three hundred and forty five thousand, six hundred and seventy eight'.

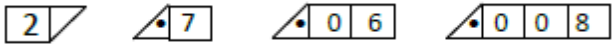
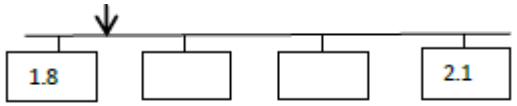
Identify rows as ten times the one above and a tenth of the one below. Relate the position of digits to multiplying or dividing by 10.
Order and compare whole numbers, identifying the value of digits.
Round numbers to the nearest 10 or 100. Read values from a variety of scales.

Counting and properties of numbers

Count on and back from any number in steps of different size, noting patterns in the sequences:

- steps 3, 4, 8, 20 and 50, recognising multiples
- steps of different unit fractions
- discuss how to bridge multiples of 10 and relate this to strategies for adding and subtracting.

Estimate and then count a larger set of objects (up to 100), grouping them as appropriate, e.g. in tens.
Write and spell three digit numbers in words accurately.

Stage 4	Stage 5	Stage 6
Place value, ordering and rounding Place value, ordering and rounding Extend the place value chart to build the language of numbers to three places of decimals: 0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90 100 200 300 400 500 600 700 800 900 Work first on the language of the new rows: • work across the rows, e.g. 0.07 is 'nought point nought seven' (or 'zero') • combine numbers from the rows, e.g. 23.456 is 'twenty three point four five six'. • Write and partition decimal numbers, e.g. use arrow cards, 2.768 = 2 + 0.7 + 0.06 + 0.008. Relate the value of each digit to the pv chart, e.g. '6' is 6 hundredths.  Understand relationships between rows in the pv chart. Relate the position of digits to multiplying or dividing by 10 or 100. Round whole numbers to the nearest 10, 100 or 1000 and use to approximate answers to calculations. Read scales, including simple decimal scales. **Counting and properties of numbers** Count on and back from any number in steps of different size, noting patterns in the sequences: • using familiar steps, move into negative numbers • steps of 6, 7, 9 and 25, recognising multiples • decimal & fraction steps, e.g. 0.1, 0.2, 0.5, 0.01 1/7 2/7 3/7 etc. Write and spell four digit numbers accurately.	Place value, ordering and rounding Read, write and partition whole numbers to a million and beyond and decimals to 3 places. Understand the language of decimals in the context of units of measure, recognising money as a special case, e.g. 'three pounds twenty four'. Multiply and divide whole numbers and decimals by 10, 100 and 1000. Identify the most significant digit(s) in a number and use to order a set of numbers, including decimals with the same number of decimal places. Round whole numbers to the nearest 10, 100, 1000 and decimal numbers to nearest whole number. Use rounding to approximate answers to calculations. Read whole number and decimal scales, interpolating values. Represent negative numbers on a number line and use them in context. **Counting and properties of numbers** Maintain skills in counting with a step, including multiples of single digit numbers and powers of 10 (e.g. steps of 0.8, 8, 80, 800 or 8000) and other simple decimals and fractions, e.g. 0.25, 0.75. ¾ 1 ½ 2 ¼ 3 Use multiple representations (e.g. multiplication squares, number grids and arrays) to identify properties of numbers, including: • sum and difference of odd and even numbers • multiples, factors, squares and primes Write and spell five digit numbers and tenths and hundredths accurately.	Place value, ordering and rounding Multiply or divide any number by a power of 10 and relate to conversion between units of measure. Order a set of decimal numbers, using trailing zeros for clarity, e.g. 0.30 > 0.25. Round whole numbers to a specified power of 10 and decimals to 1 or 2 decimal places. Use rounding to approximate the answer to a calculation or to specify a range in which it lies. Construct, complete, and read from a variety of scales, interpolating values.  Use negative numbers in context and calculate intervals between two values, e.g. an increase in temperature from -7°C to -3°C or from -4°C to 5°C. **Counting and properties of numbers** Use multiple representations to identify properties of numbers, including: • products of odd and even numbers • common multiples and common factors • prime factors of numbers to 100 • patterns of square, cube and triangular numbers  Write and spell six digit numbers and decimals accurately.

Addition

Stage 1

Explore addition as:

- combining two sets to make a total ('count all')
- adding one set to another ('count on')

Finding one more of any number up to 10 and then up to 20.

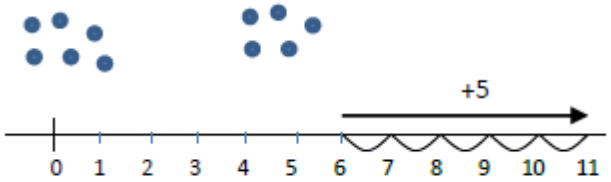
Generate partitions of 5:
5 + 0, 4 + 1, 3 + 2,
2 + 3, 1 + 4, 0 + 5.
Record related facts using plus, minus and equals ('is the same as') signs, e.g. 5 = 3 + 2, 5 = 2 + 3, 5 - 3 = 2, 5 – 2 = 3, etc.
Find complements to 5: 2 + ? = 5, etc.

Extend the above to numbers up to 10.

Finding one more of any number up to 10 and then 20.

Solve addition problems with small numbers (up to 20) by 'counting all' and then by 'counting on', e.g. using counters, linking cubes or a prepared number line. Use associated language.

I started with £6 in my money box and then collected another £5. How much money do I have now?



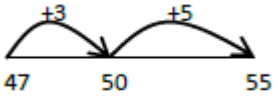
Stage 2

Relationships & facts (up to 10, multiples of 10)
Recognise addition and subtraction as inverse operations and record related facts: 6 + 4 = 10, 4 + 6 = 10, 10 – 4 = 6, 10 – 6 = 4.
When fluent with number pairs that sum to 10, extend to multiples of 10 and 100, e.g. 100 = 60 + 40, 1000 = 600 + 400 and related facts.
Find complements, e.g.70 + ? = 100 and
700 + ? = 1000

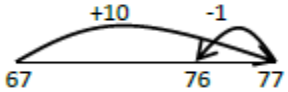
Mental methods (initially TU + U and TU + T)
Use place value to add ones or tens, e.g. 32 + 5, 46 + 30. Extend to simple cases of TU + TU, e.g. 52 + 26 = 52 + 20 + 6. (Show on '1 to 100' square and encourage use of number line to show jottings.)

Bridge across multiples of 10 (NB link to counting with a step), modelling on an empty number line:

- partition second number e.g. 47 + 8 +3 +5



- round and adjust e.g. 67 + 9 +10 -1



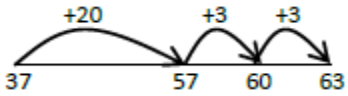
- use near doubles, e.g. 30 + 31 is double 30 plus 1

Use knowledge that addition can be done in any order, e.g. put the larger number first or add 3 or more small numbers by pairing them up into easy pairs e.g. doubles, near doubles, bonds to 10 etc.
Solve investigations and 1-step problems, moving on from counting to mental methods of addition. Begin to check using a different method.

Stage 3

Relationships & facts (up to 100)
Develop fluency with addition and subtraction facts to 10 and related multiples of 10 and 100. Extend to number pairs that partition 20.
Find complements to 100: 53 + ? = 100 (use '1 to 100' square)
Mental methods (TU + TU and HTU + TU, not bridging 100)
Secure strategies to bridge across multiples of 10:

- partition second number e.g. 37 + 26



- round and adjust , e.g. 45 + 39 = 45 + 40 - 1
- use near doubles, e.g. 23 + 25 = 2 x 24

Select a method appropriate to the numbers and explain it, e.g. by recording on an empty number line. Use knowledge that addition can be done in any order.

Written (column) methods (TU+TU, then TU + TU +TU)
Partition *both* numbers, adding tens first

	6	5	←	60	+	5		
+	7	8	←	70	+	8		
1	4	3	←	130	+	13	=	143

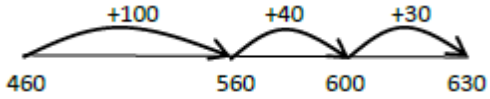
Partition to add 100+30+10+3
Extend by adding 3 two-digit numbers.

Solve investigations and 1-step problems, deciding on the operation and beginning to use written methods. Check by adding in a different order or using an alternative method.

Stage 4

Relationships & facts (up to 100)
Maintain fluency with addition and subtraction facts to 10, 20 and 100, including deriving rapidly complements to 100: 37 + ? = 100

Mental methods (3 or 4 digits, multiples of 10 or 100)
Develop fluency in selecting appropriate methods for 2-digit additions, e.g. partition, round and adjust or use near doubles, with less need for recording.
Extend to bigger numbers, particularly multiples of 10 and 100, e.g. 460 + 170, 6700 + 3800, etc.



Written (column) methods (HTU)
Establish: estimate – calculate - check
589 + 362 ≈ 600 + 350 = 950
Partition both numbers, using the language of place value and adding hundreds first:

589 & 500 + 80 + 9
+362 +300 + 60 + 2
800 800+ 140+11 = 951
140
11
951

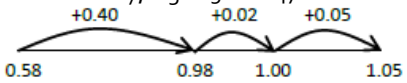
Extend to adding 3 or more numbers, each with up to 3 digits.

Solve investigations and 2-step problems, decide on and explain the operations and methods of calculation and begin to estimate and check the answer.

Stage 5

Relationships & facts (higher multiples & decimals.)
Recognise related partitions of 1000, 100, 1, 0.1 and 0.01, e.g. 300 + 700, 30 + 70, 3 + 7, 0.3 + 0.7, 0.03 + 0.07, and their families of facts.
Derive rapidly complements such as 630 + ? = 1000 and extend to decimals, e.g. 6.3 + ? = 10 and 0.63 + ? = 1, reading decimal digits correctly (use '0.01 to 1' square)

Mental methods
Continue to develop methods for larger numbers and 3 or more smaller numbers, recognising special cases, e.g. 784 + 295 (round), 564 + 320 (564 + 300 + 20), 688 + 692 (double 690), 80 + 81 + 85 + 87 (80 x 4 + 1 + 5 +7)
Extend methods to decimals, e.g. 4.6 + 2.8, 0.5 + 0.64, using 'trailing' zeros for clarity, e.g. 0.58 + 0.47:



Written (column) methods (ThHTU & decimals)
Estimate – calculate – check
7648 + 1486 ≈ 7500 + 1500 = 9000

	7	6	4	8 ←	7000	+	600	+	40	+	8				
		5	7	2	0	+	500	+	70	+	2				
+	1	4	8	6	1000	+	400	+	80	+	6				
	9	7	0	6	8000	+	1500	+	190	+	16	=	9706		

Extend to decimals

	6	2.	5	8	60	+	2	+	.5	+	.08				
+	1	9.	7	4	10	+	9	+	.7	+	.04				
	8	2.	3	2	70	+	11	+	1.2	+	.12	=	82.32		

Solve investigations and multi-step problems involving mixed operations, choose appropriate methods, estimate and check answers by a suitable method.

Stage 6

Relationships & facts
Maintain fluency with knowledge of addition and subtraction facts and methods for deriving them. When appropriate, extend to examples like
6300 + ? = 10 000 and 0.063 + ? = 0.1

Mental methods
Maintain fluency with mental methods of addition, including large numbers, decimals and three or more smaller numbers. Make up examples and classify them according to method of solution.

Written (column) methods
Estimate – calculate – check
Secure efficiency with column method. When appropriate, extend to examples like
67 300 + 38 400 and 0.628 + 0.286.

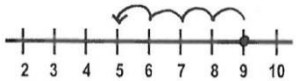
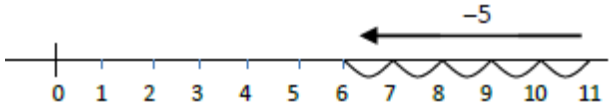
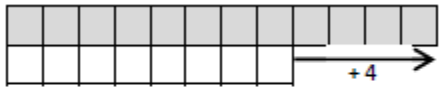
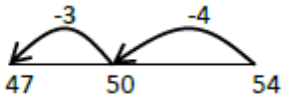
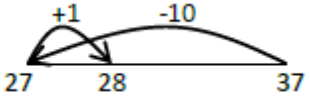
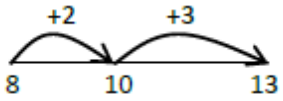
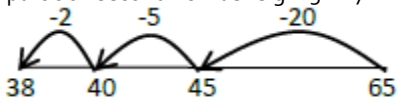
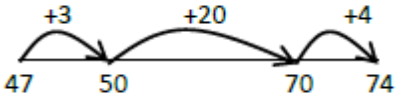
When appropriate, develop a compact method based on adding units first and using 'carry' digits, and be able to explain the connection to the expanded format:
7648 + 1486 ≈ 7500 + 1500 73.4 + 41.78 ≈ 75 + 40
≈ 9000 = 115

7648
572
+1486
9706
121

73.40
+ 41.78
115.18
1 1

Solve investigations and number problems, select and justify methods, estimate, check, use rounding and determine the level of accuracy required.

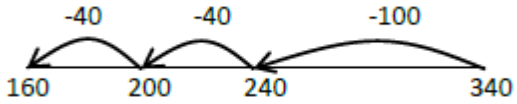
Subtraction

Stage 1	Stage 2	Stage 3
Explore subtraction as 'take away'(e.g. folded fingers, counters) and 'difference' (two rods, number line) Generate partitions of 5: 5 + 0, 4 + 1, 3 + 2, 2 + 3, 1 + 4, 0 + 5 Record a 'family' of related facts using plus, minus and equals ('is the same as') signs, 5 = 3 + 2, 5 = 2 + 3, 5 - 3 = 2, 5 – 2 = 3 Find complements to 5: 2 + ? = 5, etc. Extend the above to numbers up to 10. Use a number line to show simple subtraction.  Solve subtraction problems with small numbers (up to 20) by 'counting on' and 'counting back', e.g. using counters, linking cubes or a prepared number line. Use associated language. My sister has £11. She spends £5. How much money does she have left?  My brother has £12 and I have £8. How much more money does he have than me?  	**Relationships & facts** (up to 10, multiples of 10) Recognise addition and subtraction as inverse operations and record a family of related facts: 6 + 4 = 10, 4 + 6 = 10, 10 – 4 = 6, 10 – 6 = 4. When fluent with number pairs that sum to 10, extend to multiples of 10 and 100, e.g. 100 = 60 + 40, 1000 = 600 + 400 and related facts. Find complements, e.g.70 + ? = 100 and 700 + ? = 1000 **Mental methods** (initially TU – U and TU – T) Use place value to subtract ones or tens, e.g. 56 – 4, 56 – 20. Extend to simple cases of TU - TU, e.g. 56 – 24 = 56 – 20 – 4. (Show on a 'o to 99' square) Bridge across multiples of 10 (NB link to counting with a step), modelling on an empty number line: • partition second number e.g. 54 – 7  • round and adjust e.g. 37 – 9  • count on e.g. 13 – 8  Solve 1-step problems using mental methods. Decide when to subtract, interpreting it as both 'take away' and 'difference'. Check using addition.	**Relationships & facts** (up to 100) Develop fluency with addition and subtraction facts to 10 and related multiples of 10 and 100. Extend to number pairs that partition 20. Find complements to 100: 53 + ? = 100 (use '1 -100' square) **Mental methods** (initially TU – TU and HTU - TU not bridging 100) Secure strategies to bridge across multiples of 10: • partition second number e.g. 65 – 27 -2 -5 -20  • round and adjust, e.g. 65 – 29 = 65 – 30 + 1 • count on, e.g. 74 – 47 (see diagram below) Select a method appropriate to the numbers involved (e.g. for a small difference, count on) and explain it by recording on an empty number line. **Written (column) methods** (TU then HTU) Develop from recording of 'counting on'  74 –47 3 (to 50) 20 (to 70) 4 (to 74) 27 Solve 1-step problems, deciding on the operation and beginning to use written methods. Check by adding the answer to the number subtracted.

Stage 4

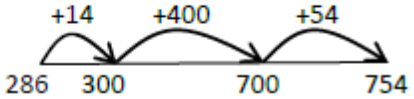
Relationships & facts (up to 100)
Maintain fluency with addition and subtraction facts to 10, 20 and 100, including deriving rapidly complements to 100:
37 + ? = 100

Mental methods (3 or 4 digits, multiples of 10 or 100)
Develop fluency in selecting appropriate methods for 2-digit subtractions, e.g. partition, round and adjust or complementary addition (count on), with less need for recording.
Extend to bigger numbers, particularly multiples of 10 and 100, e.g. 340 – 180, 5400 – 2600, etc.



Written (column) methods (HTU)
Establish: estimate – calculate – check
754 – 286 ≈ 750 – 300 = 450
Complementary addition: refine to:

754		754	
−286		−286	
14	(to 300)	14	(300)
400	(to 700)	454	(754)
54	(to 754)	468	
468			

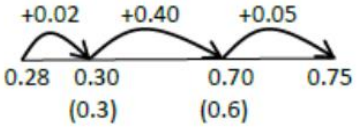


Solve 2-step problems, decide on and explain the operations and methods of calculation and begin to estimate and check the answer.

Stage 5

Relationships & facts (higher multiples & decimals)
Recognise related partitions of 1000, 100, 10, 1, 0.1 and 0.01, e.g. 300 + 700, 30 + 70, 3 + 7, 0.3 + 0.7, 0.03 + 0.07, and their families of facts.
Derive rapidly complements such as
630 + ? = 1000 and extend to decimals, 6.3 + ? = 10 and 0.63 + ? = 1, reading decimal digits correctly

Mental methods
Continue to develop methods for larger numbers, recognising special cases, e.g.
563 – 297 (round),
4007 – 3956 (count on),
336 – 68 = 338 – 70 (equivalent calculations)
Extend to decimals, e.g. 3.4 – 1.7, 0.75 – 0.28, 0.7 – 0.42, perhaps using 'trailing' zeros for clarity:
0.75 – 0.28



Written (column) methods (ThHTU & decimals)
Estimate – calculate – check
6467 – 2684 ≈ 6500 – 2500 = 4000
Refine complementary addition: extend to decimals: *Use a number line or show use of imaginary number lines*

Solve investigations and multi-step problems involving mixed operations, choose appropriate methods, estimate and check answers by a suitable method.

Stage 6

Relationships & facts
Maintain fluency with knowledge of addition and subtraction facts and methods for deriving them. When appropriate, extend to examples like
6300 + ? = 10 000 and 0.063 + ? = 0.1

Mental methods
Maintain fluency with mental methods of subtraction, including large numbers and decimals. Pupils make up examples and classify them according to method of solution.

Written (column) methods
Estimate – calculate – check
Secure efficiency with complementary addition as the agreed column method. When appropriate, extend to examples like
54 200 – 27 900 and 0.542 – 0.279.

When appropriate, enrich experience by exploring one or two other methods of written column subtraction. Pupils explain how the methods work and compare to their current established strategy.

			300									
	2000	+	400	+	^{1}50	+	6	→	2	4	5	6
-	1000	+	300	+	80	+	5	-	1	3	8	5
	1000	+	0	+	70	+	1		1	0	7	1

Solve number problems & investigations, select and justify methods, estimate, check, use rounding and determine the level of accuracy required.

Multiplication

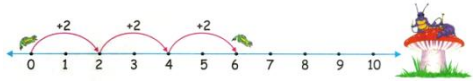
Stage 1

Use practical examples and pictorial representation of groups and arrays, e.g. how many eggs can you see?

Find doubles to 5 + 5 (e.g. use fingers)

Count in 2s: legs, arms, …
Count in 5s: fingers, toes, …
Count in 10s: fingers of both hands, …

Draw number lines and show counting in 2s, 5s and 10s.



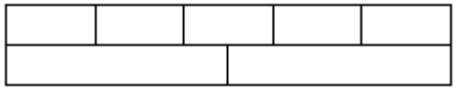
Solve simple problems by counting, e.g. using counters, pictorial representation. Begin to use associated language,
e.g. '5 lots of 2' (Maths Makes Sense language and actions - "I love what you're doing. Do it ? times."

There are 5 pairs of slippers. How many slippers altogether?

Stage 2

Use representations to show multiplication as repeated addition and that the order of multiplication does not matter, e.g. using arrays:

Represent by trains of rods (or on a number line):



Write the product either way: 2 x 5 or 5 x 2, interpreting 'x' flexibly as 'times' and 'multiplied by'
Double numbers to 10 and find corresponding halves. Use partitioning and rounding, e.g.

- double 6 is double 5 plus double 1
- double 9 is double 10 minus double 1
- half of 16 is half of 10 plus half of 6

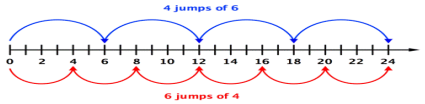
Double and halve multiples of 10 to 100 and multiples of 100 to 1000 (e.g. double 70 is double 50 plus double 20, half of 700 is half of 600 plus half of 100)
Build tables for x2 and x 10, extending to x5:

- for x5, multiply by 10 and halve, e.g.7x 5 = 7x10x½

Work with multiplication and division as inverse operations and use the signs x, **÷, =** to record families of related facts, e.g.
3 x 5 = 15, 5 x 3 = 15, 15 ÷ 5 = 3, 15 ÷ 3 = 5

Use Maths Makes Sense language X = "I love what you're doing. Do it ? times."

Show multiplication (repeated addition) on a number line.



Solve investigations and 1-step problems using materials and pictorial representation and begin to use mental methods

Stage 3

Relationships and facts
Understand multiplication as repeated addition and as scaling, e.g. My brother has £4 and I have 3 times as much. How much pocket money do I have?

Double multiples of 5 to 100 and multiples of 50 to 1000 and find corresponding halves
Understand 'halve' as x½ and as ÷2. Double numbers to 50 and find corresponding halves, e.g.

- find 2 x 37 by partitioning into 30 + 7 or 35 + 2
- find ½ x 74 by partitioning into 70 + 4 or 60 + 14
- find 98 ÷ 2 by rounding and adjusting as 100 - 2

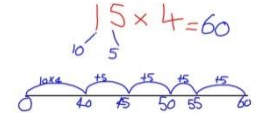
Develop fluency for x2, x5 & x10 & build tables for x3 x4 and x8:

- for x4 x8, use repeated doubling x2x2 and x2x2x2
- for x3, double & add the number, e.g. 7x 3 = 7x2+7

Mental methods (TU x U)
Use *known tables* and mental strategies to multiply 2- digit numbers up to 50 by 2, 3, 6, 4, 5 or 10:

- use place value, e.g. 40 x 10 = 400
- double and halve,

e.g. 23 x 4 = 23 x 2 x 2, 23 x 5 = 23 x10 x½
Show working out on a blank number line



- use partitioning, e.g. 38 x 3 = 30 x 3 + 8 x 3 and, to prepare for written methods, begin to develop a format for recording, e.g. 38 x 3:

x	3
30	90
8	24
	114

Solve investigations and 1-step problems (including scaling), deciding when to multiply and using mental methods

Stage 4

Relationships and facts

Double and halve numbers to 100 and beyond. Extend to related multiples of 10 and 100, e.g. 2x 370, ½x 8600

Develop fluency for x3, x4, x8; build tables for x9, x6

- for x9, multiply by 10 and subtract the number, e.g. 9 x 7 = 10 x 7 – 7
- for x6, multiply by 3 and double

Mental methods (TU x U, known tables only)

Develop a range of methods, recording as needed:

- use factors and place value
  e.g. 67 x 5 = 67 x 10 ÷2, 56 x 4 = 56 x 2 x 2
- partition (e.g. 23 x 8 = 20 x 8 + 3 x 8)
- round and adjust (e.g. 39 x 6 = 40 x 6 – 1 x 6)

Select & explain a method appropriate to a calculation.

Continue to show working on a blank number line.

Written methods (short multiplication TU x U, HTU x U)

Estimate – calculate – check

Approximate first: 365 x 8 ≈ 350 x 10 = 3500

Partitioning method, *for known tables only*.

365 x 8 = 2920

x	8
300	2400
60	480
5	40
	2920

Solve investigations and 2-step problems, decide on and explain the operations and methods of calculation and begin to estimate and check the answer.

Stage 5

Relationships and facts

Maintain skills in doubling & halving. Develop general strategies to secure fluency with tables & extend to x7:

- use commutativity, e.g. 4 x 7 = 7 x 4
- use known squares, e.g. 7 x 7 to find 8 x 7 or 6 x 7
- use partitioning, e.g. 7 x 6 = 5 x 6 + 2 x 6

Use place value to generate multiplication facts for simple decimals, e.g. 0.4 x 3, 0.02 x 6.

Mental methods (all tables)

Develop fluency in selecting appropriate methods, choosing whether to record and how:

- use factors (54 x 60 = 54 x6 x10, 28 x 7 = 4 x 7 x 7)
- partition (86 x 7 = 80 x 7 + 6 x 7)
- round and adjust (37 x 9 = 37 x 10 – 37)

Extend to multiples of 10 and 100, e.g. 470 x5, 64 x800

Estimate answers by rounding to support written methods and use of calculator.

Written methods (ThHTU x U, TU x TU, HTU x TU)

Estimate – calculate - check

637 x 38 ≈ 700 x 40 = 28000

Long multiplication:

x	30	8
600	18000	4800
30	900	240
7	210	56
	19110	5096

637 x 38 = 19110

5096

24206

Solve investigations and multi-step problems involving mixed operations, choose appropriate methods, estimate and check answers by a suitable method.

Stage 6

Relationships and facts

Continue to use strategies for rapid doubling/halving, recalling table facts and extending beyond table range.

e.g. x 12 by doubling x6, x20 by doubling x10

Mental methods

Extend to larger numbers, recognising special cases, e.g.

45 x14 = 45 x2x7, 350 x18 = 350 x 20 – 350 x 2

Extend to decimals, e.g. 3.7 x 5, 0.72 x 6.

Explore the effect of different ways of rounding when estimating answers.

Written methods (decimals, HTU x TU)

Estimate – calculate – check

Short multiplication:

3.42 x 6 ≈ 3.5 x 6

≈ 21

x	6
3	18
0.4	2.4
0.02	0.12
	20.52

Explore connections between different methods:

352 x 27 ≈ 350 x 30 = 10500

x	20	7
300	6000	2100
50	1000	350
2	40	14
	7040	2464

352

x 27

7040

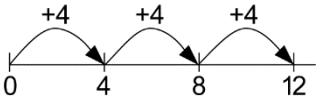
2464

9504

Pupils explain how the methods work and compare to their current established strategy.

Solve investigations and number problems, select and justify methods, estimate, check, use rounding and determine the level of accuracy required.

Division

Stage 1	Stage 2	Stage 3
Use varied contexts for practical work on sharing and grouping of objects. Solve simple problems practically using actual objects or counters. For example, 6 apples are shared equally between 2 people. How many apples does each person get?  Working practically, find a half of up to 10 objects. Discuss remainders and how they can be dealt with, e.g. 'one left over' or 'cut into two halves'.	Explore division as sharing and as grouping (and relate it to repeated subtraction): • 12 bean bags are **shared** equally into 4 buckets.  • 12 bean bags are split into buckets in **groups** of 4  Write the calculation as 12 ÷ 4 = 3 and as 12 4 = 3, regularly write division in both ways. Begin to use repeated addition using concrete objects. Calculate mentally: • divide by 2, e.g. halve numbers to 20, multiples of 10 to 200 and of 100 to 2000 • divide multiples of 10 by 10 and multiples of 100 by 10 or 100. Work with multiplication and division as inverse operations and use the signs x, **÷, =** to record families of related facts, e.g. 10 x 2 = 20, 2 x 10 = 20, 20 ÷ 2 = 10, 20 ÷ 10 = 2 Solve1-step problems (sharing and grouping) using materials and pictorial representation and begin to use mental methods. Discuss what to do with remainders, including using halves, where appropriate.	**Relationships and facts** Explore: • division as sharing or grouping (repeated subtraction) • remainders, interpreting fractional parts and some remainders which must be left whole • multiplication and division as inverse operations. For the 2, 5 and 10 times tables: • generate families of facts, e.g. 4 x 5 = 20, 5 x 4 = 20, 20 ÷ 5 = 4, 20 ÷ 4 = 5 • recognise related division facts involving multiples of 10, e.g. 6 ÷ 2 = 3, so 60 ÷ 2 = 30, 600 ÷ 2 = 300 Establish that finding a unit fraction of a number, one half (½), one fifth (1/5) and one tenth (1/10), is equivalent to dividing by 2, 5 or 10. Link to small remainders in simple practical contexts e.g. 6 sausages between 5 people gives 1 sausage and 1/5th of a sausage each. **Mental methods** (TU ÷ U, with remainders) Divide 2-digit numbers by 2, 5 and 10: • divide by 10, e.g. 60 ÷ 10, 75/10 = 7 r5, 1/10 x 500 = 50 • use halving, e.g. 78/2 = 1/2 x 78 = 1/2 x 70 + 1/2 x 8 • use related multiplication facts, e.g. for 37 ÷ 5: 7 x 5 = 35, so 37 ÷ 5 = 7 r 2 • partition larger numbers into known multiples of the divisor, e.g. 85/5 = 50/5 + 35/5 = 10 + 7 = 17 • divide by repeated addition on a number line.  12÷4=3 Solve investigations and1-step problems (sharing and grouping), deciding when to divide and using mental methods. Continue to deal with remainders.

Stage 4

Relationships and facts

Use known tables to generate families of related multiplication and division facts, e.g. 4 x 8 = 32, 8 x 4 = 32, 32 ÷ 8 = 4, 32 ÷ 4 = 8, extending e.g. to 320 ÷ 4 = 80, 3200 ÷ 4 = 800.

Mental methods (TU/U, ÷2, 5, 10, and 3, 4, 8)

Develop a range of methods, recording as needed:

- partition, using large known multiple of the divisor, e.g. 84/3 = 60/3 + 24/3 = 20 + 8
  1/8 x 96 = 1/8 x 80 + 1/8 x 16
- use partitioning to develop the use of fractions to represent remainders, e.g. 77/10 = 70/10+ 7/10 = 7710
- going beyond TU, use halving to divide by 2 or 4 and divide by 10 and 100.

Select & explain a method appropriate to a calculation.

Written methods (short division TU ÷ U, known tables)

Estimate – calculate - check

Use large multiples of the divisor to establish a range for the answer, e.g. for 86 ÷ 3, the answer lies between 20 and 30 because 60 < 86 < 90. The lower end of the range also gives the first multiple to subtract. Show working out on a number line.

```
3 ) 86
   -60   (x20)
    26
   -24   ( x8)
     2
```

Check: 3 x 28 + 2 = 84 + 2 = 86, so 86 ÷ 3 = 28 r2 or $28\frac{2}{3}$

Solve investigations and 2-step problems, decide on and explain the operations and methods of calculation and begin to estimate and check the answer, e.g. by multiplication.

Stage 5

Relationships and facts (all tables)

Generate extended division facts, e.g. 42/6 = 7, 420/6 = 70, 420/60 = 7, 4200/60 = 70, etc.

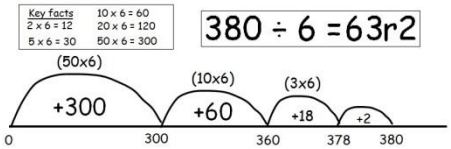
Mental methods (all tables, including ÷ 6, 7 and 9)

Develop methods, choosing when to record and how:

- partition using a large multiple of the divisor,

e.g. 92/6 = 60/6 + 32/6 = 15 r 2 or 15 26 = 15 1/3

- continue to use halving and dividing by 10 and 100, including for larger numbers



Estimate the solution range for a short division using multiples of the divisor, e.g. for 476/6 the solution is between 6 x 70 and 6 x 80, so 70 < 476/6 < 80.

Written methods (short division HTU ÷ U, ThHTU ÷ U)

Estimate – calculate - check

E.g. 4727 ÷ 6, answer will be between 700 and 800 because 4200 < 4727 < 4800. The lower end of the range also gives the first multiple to subtract.

(Encourage children to draw a number line to support these calculations)

```
6 )4727
  -4200  (x700)
   527
  -480   (x80)
    47
   -42   (x7)
     5
```

6 x 787 + 5 = 4727, so
4727 ÷ 6 = 787 r 5 or $787\frac{5}{6}$

Solve investigations and multi-step problems involving mixed operations, choose appropriate methods, estimate and check answers.

Stage 6

Relationships and facts

Develop fluency in generating extended division facts, including for decimals, e.g.

24 ÷ 6 = 4	134 ÷ 2 = 67
2.4 ÷ 6 = 0.4	13.4 ÷ 2 = 6.7
0.24 ÷ 6 = 0.04	1.34 ÷ 2 = 0.67

Mental methods

Select and compare appropriate methods and develop fluency in using them eg. dividing by 10 and then doubling answer to divide by 5.

Explore different ways to find approximations to a long division by rounding, e.g. 846 ÷ 23

≈ 800 ÷ 20 = 40 or ≈ 850 ÷ 25 = 36.

Written methods (long division HTU ÷ TU)

Estimate – calculate - check

e.g. 958 ÷ 27 ≈ 900 ÷ 30 = 30

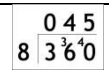
Construct a partial multiple table (using x 10, doubling and halving) Use the table to help select a large multiple to subtract (many may still wish to draw a number line to support)

27 x 1 → 27
x 2 → 54
x 4 → 108
x10 → 270
x 5 → 135

```
27)958
  540   (x20)
  418
  270   (x10)
  148
  135   (x5)
   13
```

27 x 35 + 13 = 958,
so 958 ÷ 27 = 35 r 13 or $35\frac{13}{27}$

Explore other efficient methods

		Solve investigations and problems including measures and money, where a decimal value is divided by a single digit integer. Select and justify methods, estimate and check. Interpret the remainder in context of the problem, use rounding and determine the level of accuracy required. 

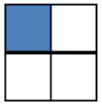
Fractions and Ratio

Stage 1

Naming, counting and ordering fractions

Explore halves and quarters in practical contexts.

Work with squares or circular shapes of different sizes. Divide into 2 or 4 equal parts and identify one of the parts as a half (1/2) or a quarter (1/4) of the shape.



Fold, shade or cut the parts and describe relationships, such as 'two halves make a whole', 'two quarters make a half', 'if I give you a quarter, three quarters are left'.

Explore body rotations and describe relationships such as 'two half turns is the same as one complete turn' 'two quarter turns is the same as a half turn'.
Learn to spell half.

Fractions as a comparison of two quantities

Compare quantities where one is half the other: the car is half as long as the van, the doll is half as tall as the teddy, this plate has half as many cookies as that plate.

Fractions as operators

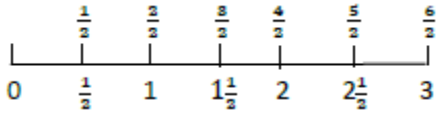
Find one half or one quarter of a quantity, such as a length or distance, or of a small number of objects.

Stage 2

Naming, counting and ordering fractions

Working on a number line:

- label 'half way' values, i.e. $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, ...
- count in halves: 'one half', 'two halves', 'three halves', ..., writing 'three halves' as 3/2
- recognise equivalences, e.g. 'three halves are the same as one and a half', 'six halves are three'.



Use Maths Makes Sense language "Same value, different appearance."
Use fractions of shapes, objects (e.g. apples) or movements, alongside the number line, to identify and describe relationships such as $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$ or $1\frac{1}{2}$.

Extend use of the number line to quarters.
Learn to spell quarter.

Fractions as a comparison of two quantities

Generate practical examples (e.g. with counters, bricks, rods or linking cubes) to compare two quantities where one is 'half', 'quarter' or 'third' of the other.

Say both ways 'The red rod is one quarter the length of the brown rod' and 'The brown rod is four times the length of the red rod.'

Fractions as operators

Understand finding 'half of', 'quarter of' and 'third of' as equivalent to dividing by 2, 4 or 3. Reinforce the equality of parts by folding lengths of string or paper strips.
Find halves of a number of items writing 'half of' as '1/2 of', e.g. 1/2 of 11 = 51/2. Find quarters of even numbers of items, '1/4 of', by halving and halving again.
Solve investigations and simple problems involving halves and quarters, including rotations and telling the time.

Stage 3

Naming, counting and ordering fractions

Using the number line image, extend from halves and quarters to work with thirds (then fifths and tenths):

- label thirds, expressing fractions greater than 1 as mixed numbers, e.g. 4 x 1/3 = 4/3 or 1 1/3
- count in thirds
- add and subtract thirds, e.g. 53 + 23 = 73 = 213

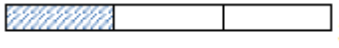
Mark up number lines and form sets of equivalent fractions by identifying different labels for the same value, e.g. {1/1, 2/2, 3/3, 4/4, 5/5, 10 /10}, {1/2, 2/4, 5 /10}, {3/2, 6/4, 15/10}, {1/5, 2/10}.
Learn to spell third, fifth, sixth, seventh, eighth, ninth and tenth.

Fractions as a comparison of two quantities

Generate practical examples and draw diagrams to compare two quantities, using 1/2, 1/3, 1/4, 1/5, 1 /10 to express one quantity as a fraction of the other. Use multiplication to describe relationships, e.g. 'one fifth as many as' (written as '1/5 x') is the inverse of 'five times as many as'
(written as '5 x').

Fractions as operators

Find 'a third of' a quantity by dividing by 3, e.g.
1/3 of 30 (1/3 x 30) is 30 divided by 3 (written as 30 / 3)



Similarly for fifths and tenths, e.g. 15 x 35 = 35 5
Find unit fractions of quantities involving halves, thirds, quarters, fifths and tenths. Use quantities which are divisible without remainder, e.g. 1 /10 x 70 = 70 /10 = 7 (except when halving, e.g. 1/2 x 9 = 9/2 = 41/2).
Add and subtract unit fractions with the same denominator within one whole.
¼ + ¼ = $^2/_4$ $_{4/10} - _{1/10} = {}^3/_{10}$
Solve simple problems and investigation involving familiar fractions, including in the context of measures.

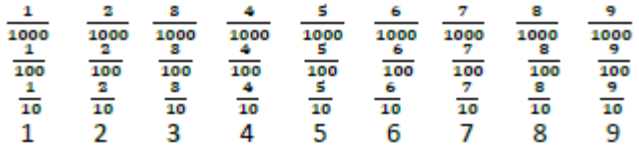
Stage 4

Naming, counting and ordering fractions

Use the number line to label, count, add and subtract fractions/mixed numbers with the same denominator. Order unit fractions; compare fractions with ½ or 1.

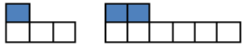
Explore sets of equivalent fractions (e.g. build fraction 'walls'), identifying the simplest in a set.

Compare fraction and decimal (place value) charts and identify decimal equivalents for 1/10, 1/100, 1/1000, 1/2, 1/5.



Fractions as comparison of two quantities

Classify diagrams and practical examples into sets where two quantities can be compared using a unit fraction, e.g. 1/3.



Recognise the multipliers, x3 and x 1/3, recording and extending to larger quantities.

Recognise and write decimal equivalents: ½ = 0.5

Fractions as operators

Find 1/2, 1/3, 1/4, 1/5 or 1/10 of quantities and relate to division with a remainder, e.g.

1/4 x 19 = 19/4 = 16/4 + 3/4 = 4 3/4 (or 4 r 3)

Use practical contexts to explain the remainder as a fraction, e.g. 3 oranges divided by 4 gives 3 quarters, 3 ÷ 4 = 34.

Link to counting: 1/4 of 1 = 1/4 … 1/4 of 2 = 24 … 14 of 3 = 34 …, etc

Add and subtract unit fractions with the same denominator

$^3/_4 + {}^3/_4 = {}^6/_4$ same value different appearance = $1\ {}^2/_4 = 1\ {}^1/_2$

Solve problems involving fractions, e.g. relating to money, metric measures and simple scaling.

Stage 5

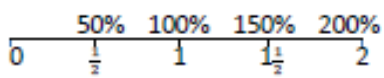
Naming, counting and ordering fractions

Generate sets of equivalent fractions and generalise by identifying common multipliers, e.g. $\frac{2}{8} = \frac{2 \times 1000}{8 \times 1000} = \frac{2000}{8000}$.

Order proper fractions, e.g. compare to 1/2 (4/9 and 7/12) or express with same denominator (2/3 and 7/9, or 5/8 and 7/12) and, in simple cases, add and subtract, e.g. 2/3 + 1/6.

Identify fraction equivalents of decimals, e.g. 0.62 = 62/100, and decimal equivalents of multiples of 1/2 , 1/4 and 1/5.

Taking whole, 100% as a find percentage equivalents for



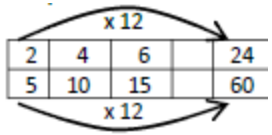
multiples of 1/2, 1/4, 1/5, 1/10, 1/100

Fractions as ratios of two quantities

Represent the relative size of two quantities (e.g. with rods or linking cubes), showing one as a non-unit fraction of the other, e.g. 25:

Use integer scaling to generate other pairs of quantities that are in the same ratio, supporting with diagrams.

Record and extend in a table.



Fractions as operators

Find non-unit fractions of quantities, e.g.

4/3 x 27 = 4 x 1/3 x 27= 4 x 9, 2/5 x 16 = 2 x 1/5 x 16 = 2 x 31/5.

For percentage operators, use fraction equivalents or multiples of 1%, 5% and 10%, e.g. 40%x£65, 125%x £40

Solve problems involving fractions, percentages and simple ratios.

Stage 6

Naming, counting and ordering fractions

Simplify fractions by identifying common factors of numerator and denominator, e.g. $\frac{20}{25} = \frac{4 \times 5}{5 \times 5} = \frac{4}{5}$.

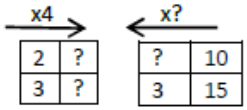
Use equivalence to order, add and subtract fractions.

Convert decimals and percentages to fractions and, where the denominator is a factor of 100, convert fractions to decimals and percentages.

Fractions as ratios of two quantities

Represent contexts using ratio notation to compare quantities, such as '2 for every 3', expressed simply as 2:3 ('two to three').

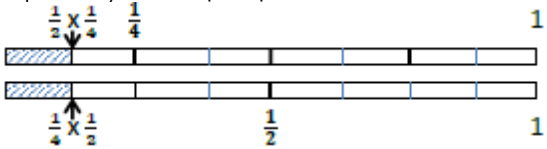
Scale quantities that are x4 x? in the same ratio (e.g. 2:3), using integer or unit fraction scale factors.



Fractions as operators

Use fractions and percentages as operators, estimating answers and comparing methods, e.g. find 40% using 2/5 x, or by adding 4 lots of 10%.

Illustrate relationships between unit fractions, e.g. use number strips to explain why 1/2 of 1/4 = 1/4 of 1/2 = 1/8:



Solve fraction and simple proportion problems, where quantities are in the same ratio, e.g. scale a recipe.